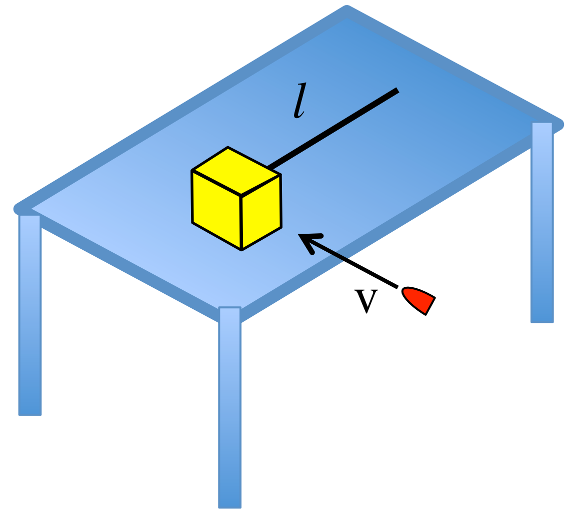
Problem 11.37

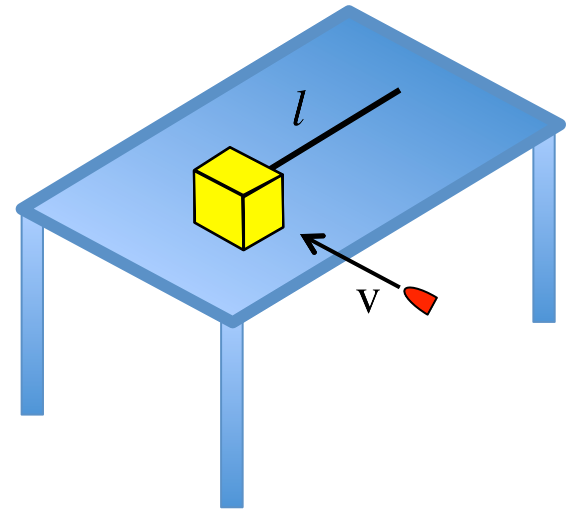


The slug applies both a force and torque on the block, and the block applies an equal and opposite force and torque back on the slug. As such, with no additional, external torques being applied, *angular momentum* will be conserved throughout. *Momentum* will be conserved through the collision even though a few instances after the collision the rod, due to its pinned end, will begin to exert an external force on the system rendering *momentum* no longer conserved thereafter.

a.) What is the initial *angular momentum* of the system?

I assume this preliminary step, which is executed on the next page, is designed to point out the fact that the slug moving in a straight line has *angular momentum*, like you didn't already know that. Anyway . . .

Initial *angular momentum* of the system?



$$
\begin{aligned}
L_o &= L_{slug} \\
&= \vec{r}_{slug} \times \left(m_{slug} \vec{v} \right) \\
&= mvl \sin 90^\circ \\
&= mvl
\end{aligned}
$$

b.) The fraction of KE lost due to the collision requires that we determine the final angular speed. To do that, our only tool is *conservation of angular momentum*. Treating the slug and mass as *point masses*:

$$\sum L_{beforeCollision} + \sum \cancelto{0}{\Gamma_{external}} \Delta t = \sum L_{afterCollision}$$

$$\Rightarrow \quad \vec{r}_{slug} \times \left(m_{slug} \vec{v} \right) = \left(I_{block} + I_{slug} \right) \omega$$

$$\Rightarrow \quad mvl = \left(Ml^2 + ml^2 \right) \omega$$

$$\Rightarrow \quad \omega = \frac{mv}{(M+m)l}$$

With the final *angular speed*, we can write:

$$KE_{initial} = \frac{1}{2} m_{slug} v^2$$

$$KE_{final} = \frac{1}{2} I_{total} \omega^2$$

$$= \frac{1}{2}\left(Ml^2 + ml^2\right)\left(\frac{mv}{(M+m)l}\right)^2$$

$$= \frac{1}{2}\left(\frac{m^2}{(M+m)}\right)v^2$$

$$\text{fraction} = \frac{\Delta KE_{final}}{KE_{initial}}$$

$$= \frac{\frac{1}{2}\left(\frac{m^2}{(M+m)}\right)v^2 - \frac{1}{2}mv^2}{\frac{1}{2}mv^2}$$

$$= \frac{\frac{1}{2}mv^2\left(\frac{m}{(M+m)}\right) - \frac{1}{2}mv^2}{\frac{1}{2}mv^2}$$

$$= \frac{\left(\frac{1}{2}mv^2\right)\left[\left(\frac{m}{(M+m)}\right) - 1\right]}{\left(\frac{1}{2}mv^2\right)}$$

$$= \left(\frac{m}{(M+m)}\right) - 1 = \left(\frac{m}{(M+m)}\right) - \frac{(M+m)}{(M+m)} = \frac{M}{(M+m)}$$